

# Compactified D=11 Supermembranes and Symplectic Non-Commutative Gauge Theories

I. Martín <sup>1 \*</sup> , J. Ovalle <sup>†</sup> and A. Restuccia <sup>2 ‡</sup>

Departamento de Física, Universidad Simón Bolívar, Venezuela  
and

<sup>1</sup> Theoretical Physics Group, Imperial College , London University

<sup>2</sup> Department of Mathematics, King's College, London University.

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It is shown that a double compactified  $D = 11$  supermembrane with non trivial wrapping may be formulated as a symplectic non-commutative gauge theory on the world volume. The symplectic non commutative structure is intrinsically obtained from the symplectic 2-form on the world volume defined by the minimal configuration of its hamiltonian. The gauge transformations on the symplectic fibration are generated by the area preserving diffeomorphisms on the world volume. Geometrically, this gauge theory corresponds to a symplectic fibration over a compact Riemannian surface with a symplectic connection.

## I. INTRODUCTION

Noncommutative geometry in string theory with a nonzero B-field [1] has recently been discussed by several authors [2]- [5]. The relation of noncommutative Yang-Mills to Born-Infeld lagrangian was considered in [6], [7] and general aspects of non-commutative gauge theories have been discussed in [8]- [22]. In [16], the change of variables from ordinary to noncommutative Yang-Mills was explicitly found and the equivalence between the Born-Infeld action for ordinary Yang-Mills in the presence of a B-field and of a noncommutative Yang-Mills was proven.

In this work, we follow a different approach. We relate the double compactified D=11 closed supermembrane [23] dual [24]- [26] to a symplectic noncommutative gauge theory on the world volume minimally coupled to seven scalar fields representing the transverse coordinates to the brane.

We first show that there is a natural symplectic structure for the double compactified supermembrane with non trivial wrapping on the target space. It is defined by the minimal configurations of the hamiltonian [25] , [26]. In fact, the solutions when interpreted in terms of connection 1-forms over principle bundles satisfy the global condition

$${}^*F(A) = n. \quad (1)$$

On the other hand, it is known that for a given a symplectic structure over a manifold there always exists a globally defined deformation of the Poisson bracket. Moreover, even for Poisson manifolds it is possible to globally define a Moyal bracket [27] which leads to a non commutative geometry. Having this idea in mind, we show that the hamiltonian for the dual of the the double compactified supermembrane corresponds exactly to a Super-Maxwell theory of a symplectic connection on a symplectic fibration. The fibre being the space generated by transverse coordinates and its conjugate momenta to the brane in the target phase space. It is noticed that a deformation of the given geometrical structure in this theory will lead in a straightforward way to a non commutative (à la Moyal) gauge theory. The reformulation of the compactified  $D = 11$  supermembrane dual in terms of noncommutative gauge theories provides a different point of view to analyze fundamental properties of the supermembrane as discussed in [28], [29].

The steps taken in our formulation are as follow: we first construct the Hamiltonian for the doubly compactified supermembrane dual. The Hamiltonian minima are smooth configurations corresponding to  $U(1)$  connections globally defined over the brane world volume. The curvature of these connections is a non degenerate 2-form that give rise to a well defined symplectic structure. The second step in our construction is then to introduce symplectic connections with its covariant derivatives in the compactified directions. The Hamiltonian reduces then to an exact symplectic non commutative Super Maxwell theory interacting with scalar fields.

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\*isbeliam@usb.ve ;isbeliam@ic.ac.uk

†jovalle@usb.ve

‡arestu@usb.ve

## II. HAMILTONIAN FORMULATION OF THE DOUBLE COMPACTIFIED $D = 11$ SUPERMEMBRANE DUAL

We consider the compactified  $D = 11$  closed supermembrane dual obtained in [24] and [26]. The bosonic part of its action is given by

$$S(\gamma, X, A) = -\frac{1}{2} \int_{\Sigma x R} d^3 \xi \sqrt{-\gamma} \left( \gamma^{ij} \partial_i X^m \partial_j X^n \eta_{mn} + \frac{1}{2} \gamma^{ij} \gamma^{kl} F_{ik}^r F_{jl}^r - 1 \right) \quad (2)$$

where  $X^m$ ,  $m = 1, \dots, D - q$  denote the maps from the world volume  $\Sigma x R$  to the target space  $M_{D-q} x \underbrace{S^1 x \dots x S^1}_q$ ,  $\Sigma$  been a compact (closed) Riemann surface.  $A_i^r$ ,  $r = 1, \dots, q$  denotes the components of the  $q$   $U(1)$  connection 1-forms over  $\Sigma x R$ .  $\gamma^{ij}$  is the auxiliary metric. We will be interested in the cases  $q = 1$  and  $q = 2$  the single and the double compactified case. The case  $q = 0$  correspond to the supermembrane action over  $M_{11}$ . The action (2) for the  $q = 1$  case is dual to the supermembrane with target space  $M_{10} x S^1$ , while the action for  $q = 2$  is dual to the supermembrane with target space  $M_9 x S^1 x S^1$ . The equivalence between the actions under duality transformations is valid off-shell. The functional integral formulations may be proven to be formally equivalent in both cases.

To obtain the hamiltonian formulation of the theory, we consider in the usual way, the ADM decomposition of the metric

$$\begin{aligned} \gamma_{ab} &= \beta_{ab} & \gamma^{ab} &= \beta^{ab} - N^a N^b N^{-2} \\ \gamma^{0a} &= N^a N^{-2} & \gamma_{0a} &= \beta_{ab} N^b \\ \gamma_{00} &= -N^2 + \beta_{ab} N^a N^b & \gamma^{00} &= -N^{-2} \end{aligned} \quad (3)$$

and define  $\beta^{ab}$  by  $\beta^{ab} \beta_{bc} = \delta_c^a$

The light-cone gauge fixing conditions are

$$X^+ = P_0^+ T \quad P^+ = P_0^+ \sqrt{W} \quad (4)$$

where  $T$  is the time coordinate on the world volume and  $W$  the determinant of the metric over  $\Sigma$  introduced through the gauge fixing condition only.

After elimination of  $X^-$  and  $P^-$  one obtains [26] the hamiltonian density

$$\mathcal{H} = \frac{1}{2} \frac{1}{\sqrt{W}} \left( P^M P_M + \beta + \frac{1}{2} \beta \beta^{ac} \beta^{bd} F_{ab}^r F_{cd}^r \right) - A_0^r \phi_r + \Lambda \phi \quad (5)$$

where  $A_0^r$  and  $\Lambda$  are the Lagrange multipliers associated to the first class constraints

$$\phi_r \equiv \partial_a \Pi_r^a = 0 \quad (6)$$

$$\phi \equiv \epsilon^{ab} \partial_b \left[ \frac{\partial_a X^M P_M + \Pi_r^c F_{ac}^r}{\sqrt{W}} \right] = 0 \quad (7)$$

$\phi$  being the generator of the area preserving diffeomorphism. There is also a global constraint arising from the elimination of  $X^-$ , this is

$$\oint_c \left( \frac{\partial_a X^M P_M + \Pi_r^b F_{ab}^r}{\sqrt{W}} \right) d\xi^a = 2\pi n_c \quad (8)$$

where  $c$  is a basis of homology of dimension one.  $n_c$  are integers associated to  $c$ .

$\beta_{ab}$  is the auxiliarly metric satisfying

$$\beta_{ab} = (1 - \beta^{-1} \Pi_r^a \Pi_r^b \beta_{ab})^{-1} (\partial_a X^M \partial_b X_M - \beta^{-1} \Pi_r^c \Pi_r^d \beta_{ca} \beta_{db}) \quad (9)$$

where  $\beta$  is the determinant of the matrix  $\beta_{ab}$ .

$P_M$  are the conjugate momenta associated to  $X^M$ . The index  $M$  refer to the transverse coordinates in the light-cone decomposition of the target space. (7) and (8) arise from the integrability condition on the resolution for  $X^-$  and the further assumption that  $X^-$ , winds up over  $S^1$  with winding numbers  $n_c$ .

It is interesting to notice that the hamiltonian density (5) depends on the auxiliarly metric only through its determinant  $\beta$ . In fact

$$\beta F_{ab}^r F_{cd}^r \beta^{ac} \beta^{bd} = \frac{1}{2} W (^*F^r)^2 \quad (10)$$

where

$$^*F^r \equiv \frac{\epsilon^{ab}}{\sqrt{W}} F_{ab}^r \quad (11)$$

is the Hodge dual to the curvature 2-form  $F^r$ .

The determinant  $\beta$  may be obtained from (9) after some calculations, it has the following expressions

$$\beta = \det(\partial_a X^M \partial_b X_M); \quad M = 1, \dots, 9 \quad (12)$$

for the  $q = 0$  case [28],

$$\beta = \det(\partial_a X^M \partial_b X_M) + (\Pi^a \partial_a X^M)^2; \quad M = 1, \dots, 8 \quad (13)$$

for the  $q = 1$  case and

$$\beta = \det(\partial_a X^M \partial_b X_M) + (\Pi_r^a \partial_a X^M)^2 + \frac{1}{4} (\Pi_r^a \Pi_s^b \epsilon_{ab} \epsilon^{rs})^2; \quad r = 1, 2; \quad M = 1, \dots, 7 \quad (14)$$

for the  $q = 2$  case.

The hamiltonian densities obtained after replacing (13) and (14) into (5) may also be constructed in a more direct way from the hamiltonian density of the supermembrane in the LCG by using duality in the canonical approach directly without starting from the covariant formulation. Let us analyze briefly this point. We consider the canonical action of the supermembrane in the LCG [28] with target space  $M_9 \times S^1 \times S^1$ , its bosonic part is

$$\mathcal{H}_{SM} = \frac{1}{2} \frac{1}{\sqrt{W}} (P^M P_M + \det(\partial_a X^M \partial_b X_M)) + \Lambda \epsilon^{ab} \partial_b \left( \frac{\partial_a X^M P_M}{\sqrt{W}} \right); \quad M = 1, \dots, 9 \quad (15)$$

Consider, first, one of the compactified coordinates taking values over  $S^1$ . It must satisfy

$$\oint_c dX = 2\pi n_c \quad (16)$$

The terms involving that map in the canonical action are

$$\langle P \dot{X} - \frac{1}{2} \frac{1}{\sqrt{W}} (P^2 + \epsilon^{ac} \epsilon^{bd} \partial_a X \partial_b X \partial_c X^N \partial_d X^N) + \partial_b \Lambda \epsilon^{ab} \partial_a X \cdot \frac{P}{\sqrt{W}} \rangle \quad (17)$$

where  $X^N$  is different from  $X$ . We may then construct an equivalent constrained term

$$\langle P L_0 - \frac{1}{2} \frac{1}{\sqrt{W}} (P^2 + \epsilon^{ac} \epsilon^{bd} L_a L_b \partial_c X^N \partial_d X^N) + \partial_b \Lambda \epsilon^{ab} L_a \frac{P}{\sqrt{W}} \rangle \quad (18)$$

subject to

$$\epsilon^{ca} \partial_c L_a = 0; \quad \partial_a L_0 - \partial_0 L_a = 0 \quad (19)$$

We may introduce them into the action (17) through the use of Lagrange multipliers, which we will denote  $A_0$  and  $\epsilon^{ab} A_b$  respectively. We then recognize that the conjugate momenta to  $A_b$  is

$$\Pi^b \equiv \epsilon^{ab} L_a \quad (20)$$

After the elimination of  $L_0$  we get

$$p = \epsilon^{ab} \partial_a A_b \quad (21)$$

(17) subject to (19) reduces then to

$$\langle \Pi^b \dot{A}_b - \frac{1}{2} \frac{1}{\sqrt{W}} [(\epsilon^{ab} \partial_a A_b)^2 + \Pi^c \Pi^d \partial_c X^N \partial_d X_N] - A_0 \partial_c \Pi^c - \Lambda \Pi^b \partial_b \left( \frac{\epsilon^{cd} \partial_c A_d}{\sqrt{W}} \right) \rangle \quad (22)$$

The terms (22) contribute together with the terms independent of  $X$  and  $P$  in (15) to give exactly the same expression of the hamiltonian density (5) and (13):

$$\begin{aligned} \mathcal{H}_D = & \frac{1}{2} \frac{1}{\sqrt{W}} \left( P^M P_M + \det(\partial_a X^M \partial_b X_M) + (\Pi^a \partial_a X^M)^2 + \frac{1}{4} W (*F)^2 \right) \\ & - A_0 \partial_c \Pi^c + \Lambda \epsilon^{ab} \partial_b \left( \frac{\partial_a X^M P_M + \Pi^c F_{ac}}{\sqrt{W}} \right); \quad M = 1, \dots, 8 \end{aligned} \quad (23)$$

(23) is also the hamiltonian density arising from the canonical formulation of the Born-Infeld action [30], it describes then the  $D2$ -brane in 10 dimensions in the case of open supermembranes. If we now repeat the above procedure for the second compactified coordinate we obtain the following hamiltonian density

$$\begin{aligned} \mathcal{H} = & \frac{1}{2} \frac{1}{\sqrt{W}} \left( P^M P_M + \det(\partial_a X^M \partial_b X_M) + (\Pi_r^a \partial_a X^M)^2 + \frac{1}{4} (\Pi_r^a \Pi_s^b \epsilon_{ab} \epsilon^{rs})^2 \right. \\ & \left. + \frac{1}{4} W (*F^r)^2 \right) - A_0^r \partial_c \Pi_r^c + \Lambda \epsilon^{ab} \partial_b \left( \frac{\partial_a X^M P_M + \Pi_r^c F_{ac}^r}{\sqrt{W}} \right) \end{aligned} \quad (24)$$

in complete agreement with (5) and (14) which were obtained from the canonical analysis of the covariant formulation of the theory. The equivalence between  $\mathcal{H}_{SM}$ ,  $\mathcal{H}_D$  and  $\mathcal{H}$  may be then established from the duality equivalence between the covariant formulations of the theories, or more directly from the duality equivalence of the gauge fixed canonical formulations in the LCG. The relation becomes non-trivial because the procedure of going from the covariant formulation to the LCG one involves the elimination of the auxiliarly metric which is an on-shell step while the duality equivalence are off-shell ones, they can be formally performed on the functional integral.

### III. THE MINIMAL CONFIGURATIONS OF THE HAMILTONIAN

We will now analyze more in detail (24). Its supersymmetric extension may be obtained in an straightforward way from the supermembrane hamiltonian in the LCG by the procedure described above, we will write the resulting expression at the end of the analysis. We may solve explicitly the constraints on  $\Pi_r^c$  obtaining

$$\Pi_r^c = \epsilon^{cb} \partial_b \Pi_r; \quad r = 1, 2 \quad (25)$$

Defining the 2-form  $\omega$  in terms of  $\Pi_r$  as

$$\omega = \partial_a \Pi_r \partial_b \Pi_s \epsilon^{rs} d\xi \wedge d\xi', \quad (26)$$

the condition of non trivial membrane winding imposes a restriction on it, namely

$$\oint_{\Sigma} \omega = 2\pi n. \quad (27)$$

With this condition on  $\omega$ , Weil's theorem ensures that there always exist an associated  $U(1)$  principal bundle over  $\Sigma$  and a connection on it such that  $\omega$  is its curvature. The minimal configurations for the hamiltonian (24) may be expressed in terms of such connections.

In [26] the minimal configurations of the hamiltonian of the double compactified supermembrane were obtained. In spite of the fact that the explicit expression (24) was not then obtained, all the minimal configurations were found. They correspond to  $\Pi_r = \hat{\Pi}_r$  satisfying

$${}^* \hat{\omega} = \epsilon^{ab} \partial_a \hat{\Pi}_r \partial_b \hat{\Pi}_s \epsilon^{rs} = n \sqrt{W} \quad n \neq 0 \quad (28)$$

The explicit expressions for  $\hat{\Pi}_r$  were obtained in that paper [26]. As mentioned before, they correspond to  $U(1)$  connections on non trivial principle bundles over  $\Sigma$ . The principle bundle is characterized by the integer  $n$  corresponding to an irreducible winding of the supermembrane [25]. Moreover the semiclassical approximation of the hamiltonian density around the minimal configuration, was shown to agree with the hamiltonian density of super Maxwell theory on the world sheet, minimally coupled to the seven scalar fields representing the coordinates transverse to the world volume of the super-brane.

#### IV. THE SYMPLECTIC NON-COMMUTATIVE FORMULATION

Let us now analyze the geometrical structure of the constructed hamiltonian. We notice that the minimal configurations of the hamiltonian introduce a natural symplectic structure in the theory through the non degenerate 2-form  $\hat{\omega}$ ,

$$\hat{\omega} = \partial_a \hat{\Pi}_r \partial_b \hat{\Pi}_s \epsilon^{rs} d\xi \wedge d\xi' \quad (29)$$

Also, that  $\hat{\Pi}_r^a$  is an invertible matrix. It allows to define the metric  $W_{ab}$  on the world volume.

$$W_{ab} = 2\partial_a \hat{\Pi}_r \partial_b \hat{\Pi}_r \quad (30)$$

Its determinant takes the value

$$\det W_{ab} = n^2 W, \quad (31)$$

and its inverse is given by

$$n^2 W^{ab} = \frac{\epsilon^{ac}}{\sqrt{W}} \frac{\epsilon^{bd}}{\sqrt{W}} W_{cd} = \frac{2}{\sqrt{W}} \hat{\Pi}_r^a \hat{\Pi}_r^b \quad (32)$$

Furthermore, we introduce the covariant derivative  $D_a$  with respect to this metric  $W_{ab}$ , it then follows that

$$D_a W = 0; \quad D_a \hat{\Pi}_r^b = 0 \quad (33)$$

We now define the rotated covariant derivatives in terms of tangent space coordinates in the compactified directions:

$$D_r \equiv \frac{\hat{\Pi}_r^a}{\sqrt{W}} D_a. \quad (34)$$

We may now perform a canonical transformation in order to introduce a symplectic connection  $\mathcal{A}_r$  in our formalism. The kinetic term

$$\langle \Pi_r^a \dot{A}_a^r \rangle \quad (35)$$

may then be rewritten as

$$\langle \Pi_r^a \dot{A}_a^r \rangle = \langle \epsilon^{ab} \partial_b A_a^r \dot{\Pi}_r \rangle = \langle \Pi^r \dot{\mathcal{A}}_r \rangle \quad (36)$$

where we have introduced

$$\Pi^r \equiv \epsilon^{ab} \partial_b A_a^r \quad (37)$$

$$\mathcal{A}_r \equiv \Pi_r - \mathcal{C}_r \quad (38)$$

where  $\mathcal{C}_r$  is a time independent geometrical object, which will be defined shortly. They satisfy the following Poisson bracket relation

$$\{\mathcal{A}_r(\xi), \Pi^r(\xi')\}_P = \delta(\xi, \xi'). \quad (39)$$

The symplectic non commutative derivative  $\mathcal{D}_r$  may be defined now as

$$\mathcal{D}_r \equiv D_r + \{\mathcal{A}_r, \quad \} \quad (40)$$

where the bracket  $\{\bullet, \bullet\}$  is defined as follows

$$\{\bullet, \bullet\} \equiv \frac{2\epsilon^{sr}}{n} D_r \bullet D_s \bullet = \frac{\epsilon^{ba}}{\sqrt{W}} D_a \bullet D_b \bullet; \quad n \neq 0 \quad (41)$$

We remark that these symplectic non commutative derivatives behave as symplectic connections on a symplectic fibration over  $\Sigma$  with the phase space  $(X^M, P^M)(\xi)$  being the fibre. The gauge transformations generated by the

first class constraint (area preserving diffeomorphisms in the base manifold  $\Sigma$ ) preserve the Poisson bracket in the fibre. The symplectic non commutative derivatives preserve, in turn, the same structure, i.e the symplectic non commutative derivatives of the fields transform under gauge transformations in the same way as the fields and the holonomies generated by the symplectic connections preserve the Poisson bracket in the fibre. These properties may be checked out by straightforward calculations. In particular,  $\delta\mathcal{A}_r = \mathcal{D}_r\xi$  under infinitesimal gauge transformations with parameter  $\xi$ .

Without loss of generality we rewrite (38) as

$$\Pi^r = \mathcal{A}_r + \hat{\Pi}_r \quad (42)$$

We then have for the terms in (24),

$$\begin{aligned} \frac{1}{\sqrt{W}}\Pi_r^a\partial_aX^M &= \frac{1}{\sqrt{W}}\hat{\Pi}_r^a\partial_aX^M + \frac{\epsilon^{ab}}{\sqrt{W}}\partial_b\mathcal{A}_r\partial_aX^M \\ &= D_rX^M + \{\mathcal{A}_r, X^M\} = \mathcal{D}_rX^M, \end{aligned} \quad (43)$$

$$det\partial_aX^M\partial_bX_M = \frac{1}{2}\partial_aX^M\partial_cX^N\partial_bX_M\partial_dX_N\epsilon^{ac}\epsilon^{bd} = \frac{1}{2}W\{X^M, X^N\}^2, \quad (44)$$

$$\begin{aligned} \Pi_r^a\Pi_s^b\epsilon_{ab}\epsilon^{rs} &= n\sqrt{W} - 2\sqrt{W}D_r\mathcal{A}_s\epsilon^{rs} + \epsilon^{bc}\partial_b\mathcal{A}_r\partial_c\mathcal{A}_s\epsilon^{rs} \\ &= n\sqrt{W} - \epsilon^{rs}\sqrt{W}(D_r\mathcal{A}_s - D_s\mathcal{A}_r + \{\mathcal{A}_r, \mathcal{A}_s\}) \\ &= (n - {}^*\mathcal{F})\sqrt{W}, \end{aligned} \quad (45)$$

where

$$\begin{aligned} {}^*\mathcal{F} &\equiv \epsilon^{rs}\mathcal{F}_{rs} \\ \mathcal{F}_{rs} &\equiv D_r\mathcal{A}_s - D_s\mathcal{A}_r + \{\mathcal{A}_r, \mathcal{A}_s\} \end{aligned} \quad (46)$$

Finally, the generator of area preserving diffeomorphisms

$$\phi \equiv \epsilon^{ab}\partial_b\left(\frac{\partial_aX^MP_M + \Pi_r^cF_{ac}^r}{\sqrt{W}}\right) \quad (47)$$

may be expressed as

$$-\phi = \mathcal{D}_r\Pi^r + \{X^M, P_M\} \quad (48)$$

The hamiltonian density (24) may then be rewritten

$$\begin{aligned} H = \int_{\Sigma}\mathcal{H} &= \int_{\Sigma}\frac{1}{2\sqrt{W}}\left[(P^M)^2 + (\Pi^r)^2 + \frac{1}{2}W\{X^M, X^N\}^2 + W(\mathcal{D}_rX^M)^2 \right. \\ &\quad \left. + \frac{1}{2}W(\mathcal{F}_{rs})^2\right] + \int_{\Sigma}\left[\frac{1}{8}\sqrt{W}n^2 - \Lambda(\mathcal{D}_r\Pi^r + \{X^M, P_M\})\right] \end{aligned} \quad (49)$$

where the following global condition has been imposed

$$\int_{\Sigma}{}^*\mathcal{F}\sqrt{W}d^2\xi = 0 \quad (50)$$

The hamiltonian (49) may be extended to include the fermionic terms of the supersymmetric theory. They may be obtained from the hamiltonian of the supermembrane in [28] by the dual approach discussed previously. They are

$$\int_{\Sigma}\sqrt{W}(\Lambda\{\bar{\theta}\Gamma_-, \theta\} - \bar{\theta}\Gamma_-\Gamma_r\mathcal{D}_r\theta + \bar{\theta}\Gamma_-\Gamma_M\{X^M, \theta\}) \quad (51)$$

where  $\theta$  is the Majorana spinor of the original formulation of the supermembrane in the LCG in D=11 which may be decomposed in terms of a complex 8-component spinor of  $SO(7)\times U(1)$ .

The hamiltonian (49) corresponds then exactly to a symplectic non-commutative super-Maxwell theory on the world volume minimally coupled to seven scalar fields  $X^M$ ,  $M = 1, \dots, 7$ . The generator of area preserving diffeomorphisms becomes the generator of gauge transformations. In distinction to the star product defined in [16] which depends on a constant large background antisymmetric field of the string which couples to the  $U(1)$  gauge fields of the D-brane, the symplectic non commutative product here is intrinsically constructed from the minimal configurations of the hamiltonian density which are unique (up to closed 1-forms) for each given  $n$  and related to the natural symplectic structure of the world volume Riemann surface. This theory may be interpreted geometrically as a symplectic fibration over a Riemann surface, with fibre given by the symplectic phase space manifold generated by the transverse coordinate to the brane in the target space. Its symplectic structure being preserved under symplectomorphism induced by the first class constraint of the theory. The connection  $\mathcal{D}_r$  is a symplectic connection on this symplectic fibration, i.e. the associated holonomies preserve the symplectic structure in the fibres [31]. Whether this symplectic fibration with a symplectic connection could be globally extended in a consistent manner to a type of Moyal non commutative gauge theory is an open question. As commented before, one can always globally deform the Poisson bracket in the fibration base space to a Moyal bracket, but it is not necessarily true that the symplectic structure on the fibre could be extended in the same way and, moreover, be preserved under holonomies.

## V. CONCLUSIONS

We have formulated the double compactified  $D = 11$  supermembrane dual with non trivial irreducible winding as a symplectic non commutative Super Maxwell theory, i.e as an exact symplectic fibration over a compact Riemann Surface with a symplectic connection. The connection dynamics being governed by a hamiltonian that resembles that of a Maxwell theory. We emphasize that our construction is globally defined. Also, we remark that the symplectic non-commutative gauge theory we have introduced relies on the nonsingular minimal configuration of the hamiltonian (24), where the assumption  $n \neq 0$  is essential. The minimal configuration obtained in [26] correspond to the monopole connection 1-forms over Riemann surfaces [32] which may also be obtained from a suitable pullback to  $\Sigma$  of the connection 1-forms on the Hopf fibring over  $CP_n$  [33]. Its curvature is a non degenerate closed 2-form defining a natural symplectic structure over  $\Sigma$ . The equivalence between the hamiltonian (24) of the double compactified  $D=11$  supermembrane dual and the hamiltonian (49) of the symplectic non-commutative geometry is exact.

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